TMDs: Global fits

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Outline

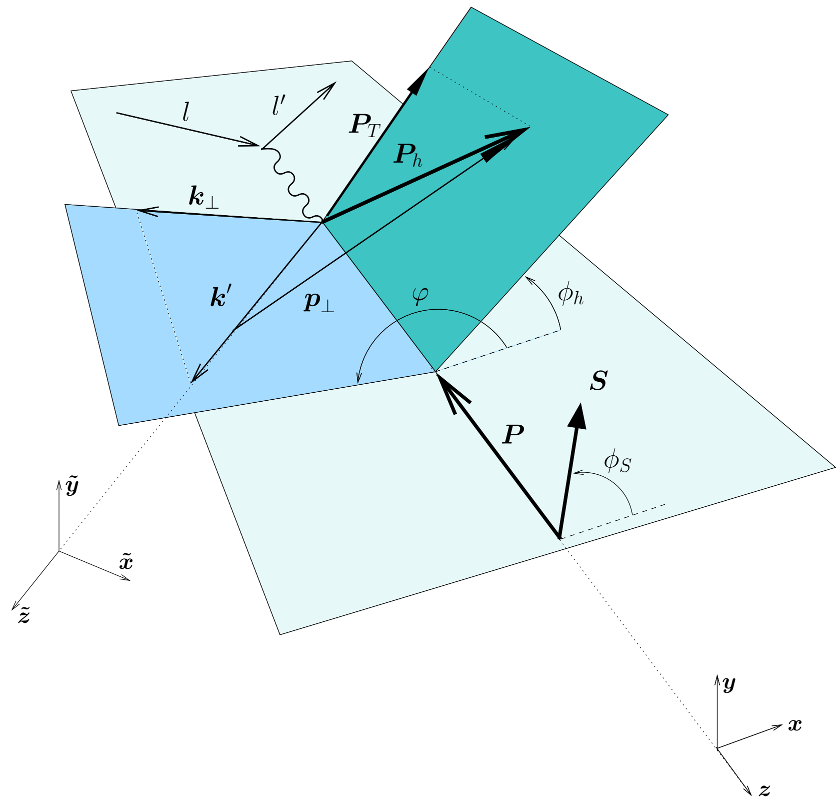
- QCD framework: TMD factorization for SIDIS & e^+e^- processes
- Data sets: HERMES, COMPASS, Belle
- Parameterizations of Sivers, Collins and transversity functions
- Global analysis and results
- A quick look at $pp \to \pi X$
- Conclusions and Open issues

SIDIS with a transversely polarized target

Transverse single spin asymmetry (SSA) in SIDIS

$$A_{UT} = \frac{d^6\sigma^{\ell p^\uparrow \to \ell' h X} - d^6\sigma^{\ell p^\downarrow \to \ell' h X}}{d^6\sigma^{\ell p^\uparrow \to \ell' h X} + d^6\sigma^{\ell p^\downarrow \to \ell' h X}} \equiv \frac{d\sigma^\uparrow - d\sigma^\downarrow}{d\sigma^\uparrow + d\sigma^\downarrow},$$

$$d^6\sigma^{\ell p^\uparrow \to \ell h X} \equiv d^6\sigma / dx_B dy\, dz_h\, d^2\boldsymbol{P}_T\, d\phi_S$$



$$\boldsymbol{A_{UT} \sim \Delta^N \hat{f}_{q/p^\uparrow} \otimes D_{h/q} \sin(\phi_h - \phi_S)}$$
$$\boldsymbol{+\ \Delta_T q \otimes \Delta^N \hat{D}_{h/q^\uparrow} \sin(\phi_h + \phi_S)}$$
$$+\cdots$$

- *different azimuthal dependences*

$\longrightarrow$ separation of **Sivers** and **Collins** effects

- access to the transversity distribution

- $k_\perp$-factorization for $\Lambda_{\rm QCD} \simeq P_T \ll Q$

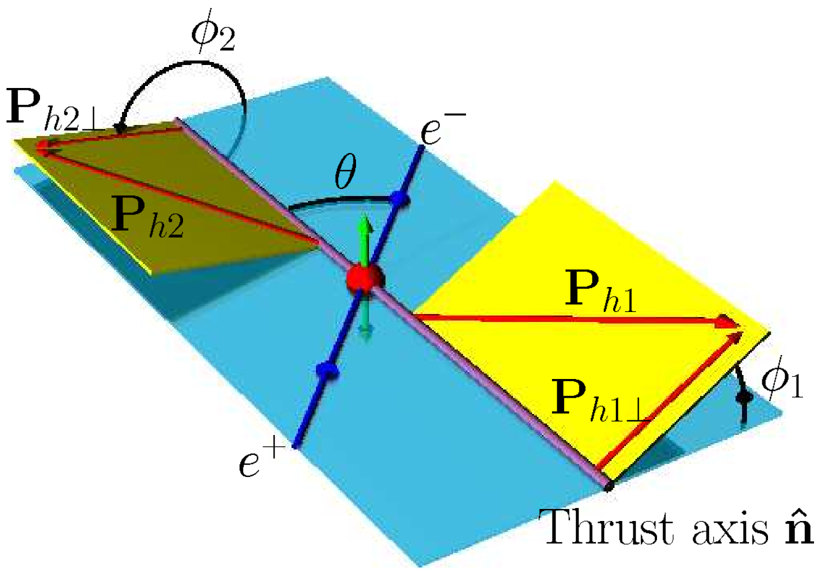
"Trento Conventions" 2004

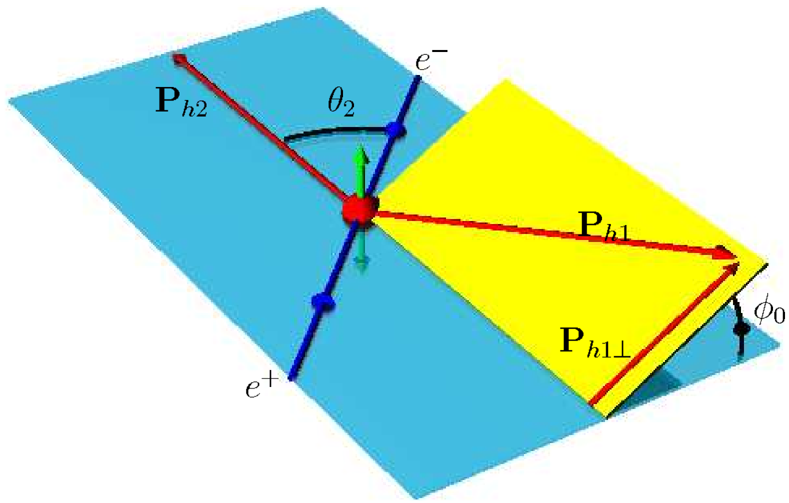
Azimuthal correlations in $e^+e^- \to h_1 h_2 + X$

(spin effects without polarization)

$$d\sigma \sim (1+\cos^2\theta)\, D_{h_1/q}\, D_{h_2/\bar{q}} + \sin^2\theta\, \Delta^N \hat{D}_{h_1/q^\uparrow}\ \Delta^N \hat{D}_{h_2/\bar{q}^\uparrow}$$

$\times \cos(\phi_1 + \phi_2)$ $\times \cos(2\phi_0)$





Boer et al. 1997

$e^+e^- \to q\bar{q} \to h_1 h_2 X$: two hadrons from opposite hemispheres in jetlike events.

Single jet event: zero result (zero $q(\bar{q})$ polarization on average) !!!

$e^+e^- \to \pi\pi\, X$ at Belle

q_T integrated: $\Delta^N D(z, p_\perp) \to \Delta^N D(z)$

$$R \equiv \frac{A_U}{A_L} \to 1 + \cos(\phi_1 + \phi_2)\, A_{12}$$

$$A_{12} = \frac{1}{4}\frac{\langle \sin^2\theta\rangle}{\langle 1 + \cos^2\theta\rangle}(P_U - P_L) \quad [\text{analogously for } A_0]$$

$A_{L/U}$: Like- and Unlike-sign pion pair yield $\to$ favoured and unfavoured FFs.

$(\Delta)D_{\rm fav} \equiv (\Delta)D_{\pi^+/u,\bar d} \quad (\Delta)D_{\rm unf} \equiv (\Delta)D_{\pi^+/d,\bar u,s,\bar s}$

$$P_U - P_L \propto \sum_q e_q^2\, \Delta^N D_{\pi/q^\uparrow}(z_1)\Delta^N D_{\pi/\bar q^\uparrow}(z_2)$$

Notice: A_{12} and A_0 data sets are NOT independent

Remarks on TMD factorization (in SIDIS, DY, e^+e^-).

- tree level

$$\sigma \simeq \int d^2\boldsymbol{k}_\perp \, d^2\boldsymbol{p}_\perp \, \delta^{(2)}(z\boldsymbol{k}_\perp + \boldsymbol{p}_\perp - \boldsymbol{P}_T) \, w(\boldsymbol{k}_\perp, \boldsymbol{P}_T, \boldsymbol{p}_\perp) f(x, \boldsymbol{k}_\perp) D(z, \boldsymbol{p}_\perp)$$

In general [*Ji et al. 2004*]

$$\simeq \int d^2\boldsymbol{k}_\perp d^2\boldsymbol{p}_\perp d^2\boldsymbol{l}_\perp \delta^{(2)}(z\boldsymbol{k}_\perp + \boldsymbol{p}_\perp + \boldsymbol{l}_\perp - \boldsymbol{P}_T) \, w(\boldsymbol{k}_\perp, \boldsymbol{P}_T, \boldsymbol{p}_\perp) f(x, \boldsymbol{k}_\perp) D(z, \boldsymbol{p}_\perp) U(l_\perp^2)$$

U: soft factor

- dilution of the asymmetry with increasing Q^2 [*Boer 2001*]
- not implemented in phenomenology: caution [*Boer 2008, Bacchetta et al. 2008*]

• Universality of the Collins function [*Metz 2002, Yuan 2008*]

Experimental data

Sivers asymmetry data

- Previous fit: *Anselmino et al. PRD72 (2005)*

HERMES data: charged pions on **proton** target

COMPASS data: charged hadrons on **deuteron** target

• information on u and d Sivers functions

- This fit:

HERMES data: charged **and neutral pions** and ***charged kaons*** on **proton** target

COMPASS data: **charged pions and charged kaons** on **deuteron** target

Improved in statistics by a factor 2

COMPASS data on **proton** target: NOT USED

• **first insight into the sea and strange Sivers functions**

Role of fragmentation function sets

Collins asymmetry data

- Previous fit: *Anselmino et al. PRD75 (2007)*

HERMES data: charged pions on **proton** target

COMPASS data: charged hadrons on **deuteron** target

Belle data: U/L ratios

• information on: u and d transversity and fav/unfav. Collins functions

- This fit:

HERMES data: charged and **neutral pions** on **proton** target

COMPASS data: **charged pions** on **deuteron** target

Belle data: U/L ratios

SIDIS data improved in statistics by a factor 2; **Belle data by a factor 20 !!!**

COMPASS data on **proton** target NOT USED

• Improved information on: u and d transversity and fav/unfav. Collins functions

Remarks:

$\langle Q^2\rangle_{\rm DIS} \simeq 2.5$ GeV^2 vs. $\langle Q^2\rangle_{e^+e^-} \simeq 110$ GeV^2 Q^2-evolution of Collins FF?

$\langle x\rangle_{\rm HERMES} \simeq 0.1$ vs. $\langle x\rangle_{\rm COMPASS} \simeq 0.03$ $\boldsymbol{x \leq 0.3}$

Sivers asymmetry:

[HERMES] proton: $A_{UT}^{\sin(\phi_h-\phi_S)} \simeq 4\,\Delta^N\hat{f}_u\,D_{h/u} + \Delta^N\hat{f}_d\,D_{h/d}$

[COMPASS] deuteron: $A_{UT}^{\sin(\phi_h-\phi_S)} \simeq (\Delta^N\hat{f}_u + \Delta^N\hat{f}_d)\,(4\,D_{h/u} + D_{h/d}) \simeq 0$

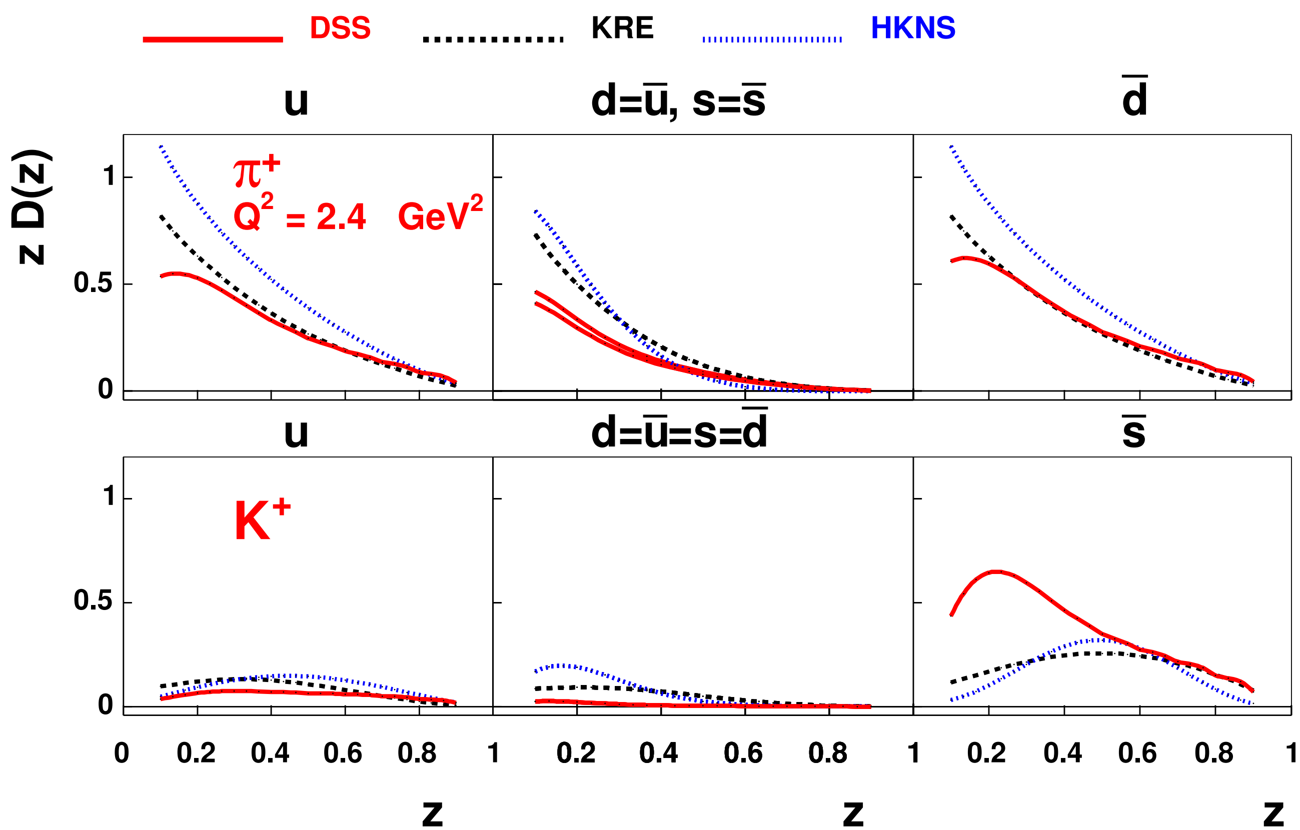
Collins asymmetry:

[HERMES] proton: $A_{UT}^{\sin(\phi_h+\phi_S)} \simeq 4\Delta_T u\,\Delta^N\hat{D}_{h/u} + \Delta_T d\,\Delta^N\hat{D}_{h/d}$

[COMPASS] deuteron: $A_{UT}^{\sin(\phi_h+\phi_S)} \simeq (\Delta_T u + \Delta_T d)\,(4\,\Delta^N\hat{D}_{h/u} + \Delta^N\hat{D}_{h/d}) \simeq 0$

Role of NEW fragmentation function set [DSS]

de Florian, Sassot, Stratmann, PRD75 (2007)



Pion and Kaon FFs at $Q^2 = 2.4$ GeV^2: large $\bar{s} \to K^+$.

Parameterizations of Sivers, Collins and transversity functions

Gaussian & factorized expressions

Sivers functions:

$$\Delta^N \hat{f}_{q/p^\uparrow}\,(x, k_\perp) = \mathcal{N}_q^S(x)\, h(k_\perp)\, 2\, f_{q/p}(x, k_\perp)$$

$$\mathcal{N}_q^S(x) \simeq N_q^S\, x^{\alpha_q}(1-x)^{\beta_q} \qquad |\mathcal{N}_q^S(x)| \leq 1$$

$$h(k_\perp) = \sqrt{2e}\,\frac{k_\perp}{M_1}\, e^{-k_\perp^2/M_1^2} \leq 1$$

$$f_{q/p}(x, k_\perp) = f_q(x)\,\frac{1}{\pi\langle k_\perp^2\rangle}\, e^{-k_\perp^2/\langle k_\perp^2\rangle} \qquad \langle k_\perp^2\rangle = 0.25\,\mathrm{GeV}^2$$

u, d and s quarks: "broken sea" ansatz fit [**11 parameters**]

N_u	N_d	N_s	$N_{\bar{u}}$	$N_{\bar{d}}$	$N_{\bar{s}}$
α_u	α_d	α_{sea}			

$\beta \iff$ **large x unconstrained** $\qquad M_1$ (GeV)

- PDF, FF sets: GRV98, DSS
- Q^2- evolution: $\Delta^N f_q$ same as f_q

Transversity and Collins functions:

$$\Delta_T q(x, k_\perp) = \mathcal{N}_q^T(x)\, \frac{1}{2}\, [f_q(x) + \Delta q(x)]\, \frac{e^{-k_\perp^2/\langle k_\perp^2\rangle}}{\pi\langle k_\perp^2\rangle}$$

$$\Delta^N \hat{D}_{h/q^\uparrow}(z, p_\perp) = \mathcal{N}_q^C(z)\, \sqrt{2e}\, \frac{p_\perp}{M}\, e^{-p_\perp^2/M^2}\, 2\, D_{h/q}(z, p_\perp)$$

$$D_{h/q}(z, p_\perp) = D_{h/q}(z)\frac{e^{-p_\perp^2/\langle p_\perp^2\rangle}}{\pi\langle p_\perp^2\rangle} \qquad \langle p_\perp^2\rangle = 0.2\,\mathrm{GeV}^2$$

$$\mathcal{N}_q^T(x) \simeq N_q^T\, x^\alpha(1-x)^\beta \quad q = u, d \qquad [4]$$

$$\mathcal{N}_q^C(z) \simeq N_q^C\, z^\gamma(1-z)^\delta \quad q = u, d, s \quad [h \equiv \pi \to \text{ fav./unfav.}] \quad [4]$$

$\rightarrow$ **9 parameters**

- PDF, FF sets: GRV98, GRSV2000, DSS
- Q^2- evolution: $\Delta_T q$ properly; $\Delta^N D_q$ same as D_q

Global fit

Uncertainties

$$\chi^2 = \sum_{i=1}^{N} \left(\frac{y_i - F(x_i; \boldsymbol{a})}{\sigma_i} \right)^2$$

- N measurements y_i at known points x_i, with variance σ_i^2.
- $F(x_i; \boldsymbol{a})$ depends *non-linearly* on M unknown parameters a_i.
- Best fit: $\chi^2_{\rm min} \rightarrow \boldsymbol{a}_0$

Error band: all sets of parameters such that $\chi^2(\boldsymbol{a}_j) \leq \chi^2_{\rm min} + \Delta\chi^2$
- $\Delta\chi^2 = 1 \leftrightarrow$ 1-σ: small errors, uncorrelated parameters, linearity, χ^2 parabolic
- $\Delta\chi^2$: fixed according to the coverage probability

$$P = \int_0^{\Delta\chi^2} \frac{1}{2\Gamma(M/2)} \left(\frac{\chi^2}{2}\right)^{(M/2)-1} \exp\left(-\frac{\chi^2}{2}\right) \mathrm{d}\chi^2 \,.$$

P= probability that true set of parameters falls inside the hypervolume

[$P = 0.68 \leftrightarrow$ 1-σ, $P = 0.95 \leftrightarrow$ 2-σ]

Sivers effect: Best fit

$\chi^2_{dof} = 1.00$ [$N_{\rm data} \simeq 170$]

χ^2 data point

pions: 0.94 vs. kaons (K^+): 1.20

unbroken-sea ansatz [8 param.]: $\chi^2_{dof} = 1.16$

χ^2 data point

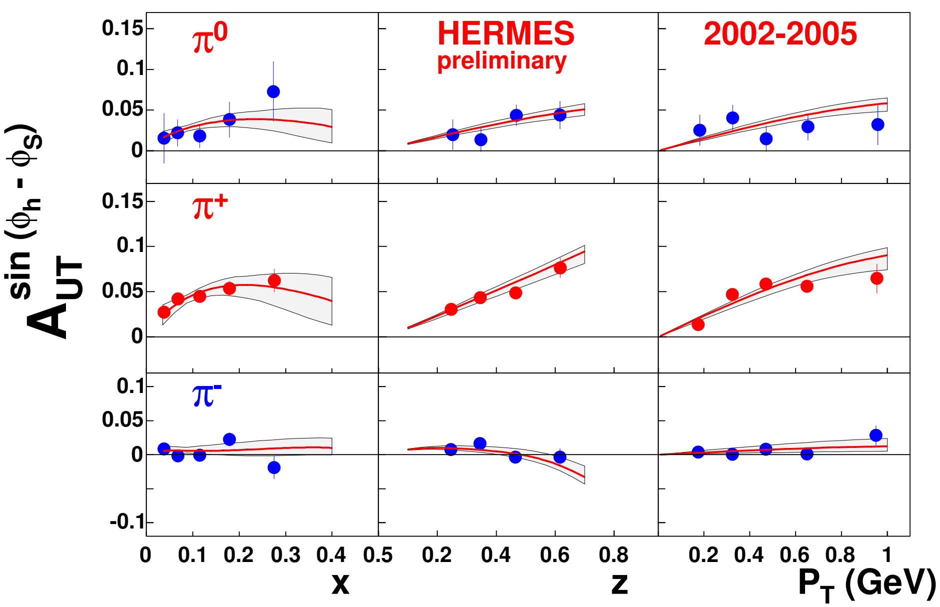
pions: $\simeq 1$ vs. kaons (K^+): $\simeq 3$

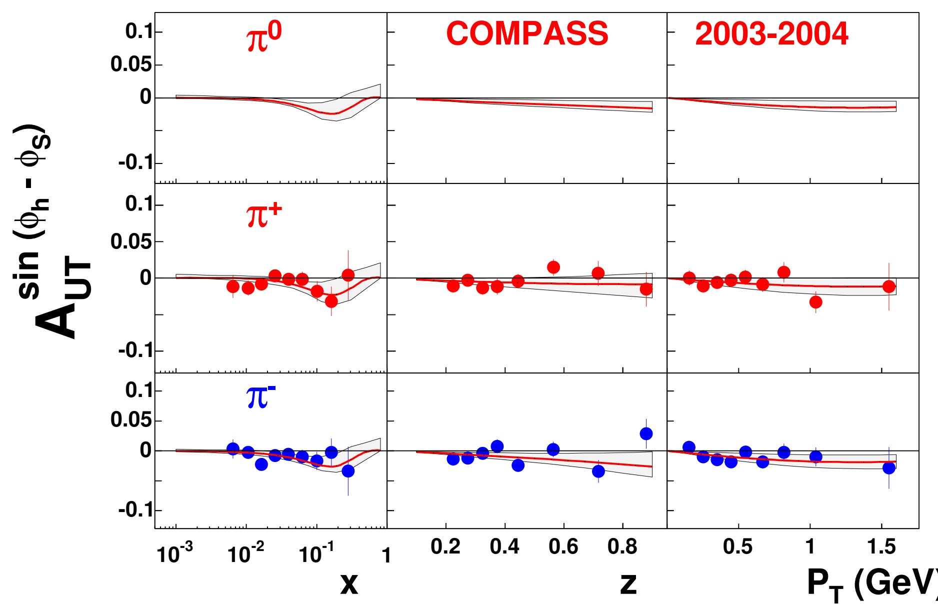
Collins effect: Best fit

$\chi^2_{dof} = 1.30$ [$N_{\rm data} \simeq 110$]

• Sivers effect in SIDIS: Best Fit

Anselmino, Boglione, UD, Kotzinian, Melis, Murgia, Prokudin, Turk, EPJA 2009

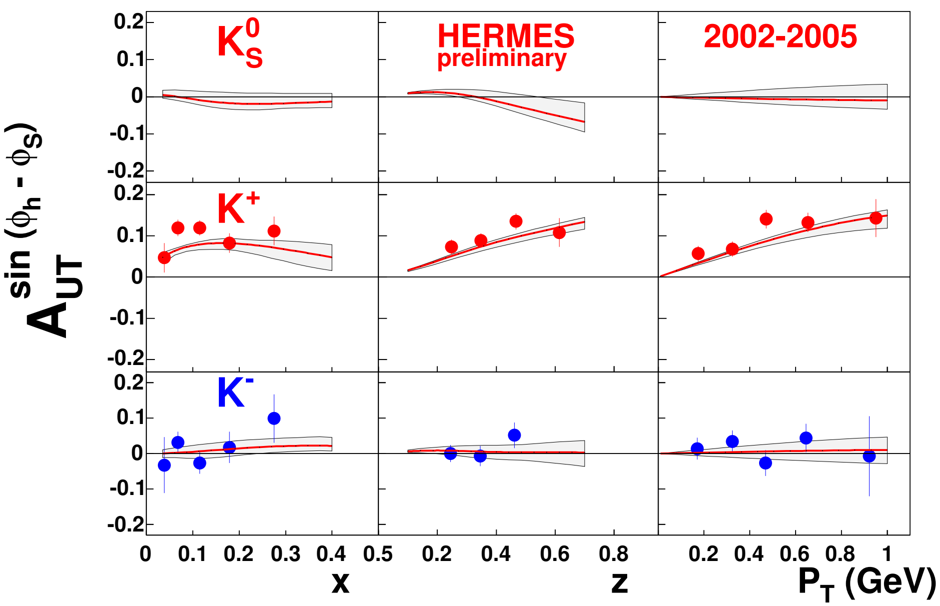


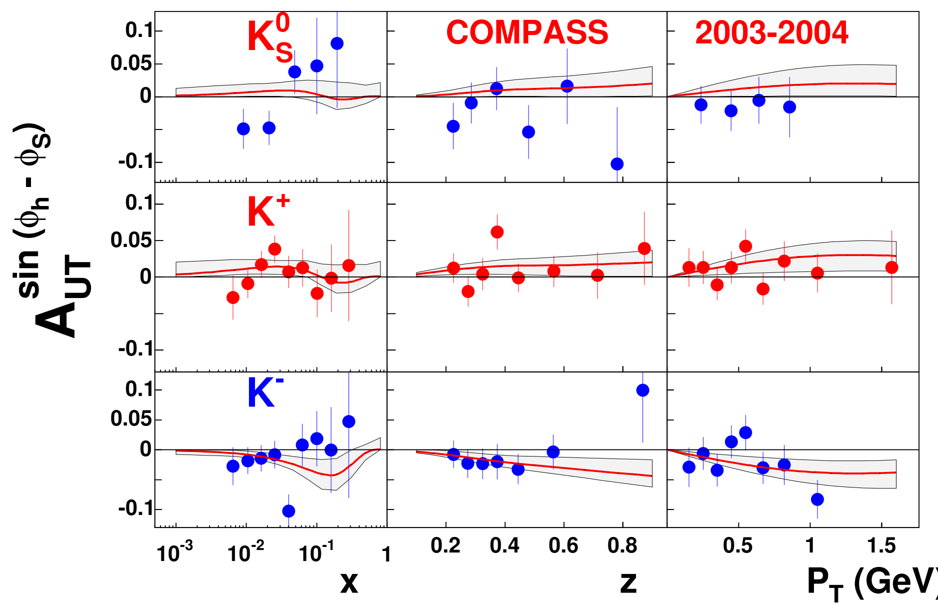


Fit of HERMES data [*Diefenthaler et al. 2006, Pappalardo et al. 2008*]

and COMPASS data [*Martin et al. 2006*] (deuteron target)

Sivers SSAs for Kaons

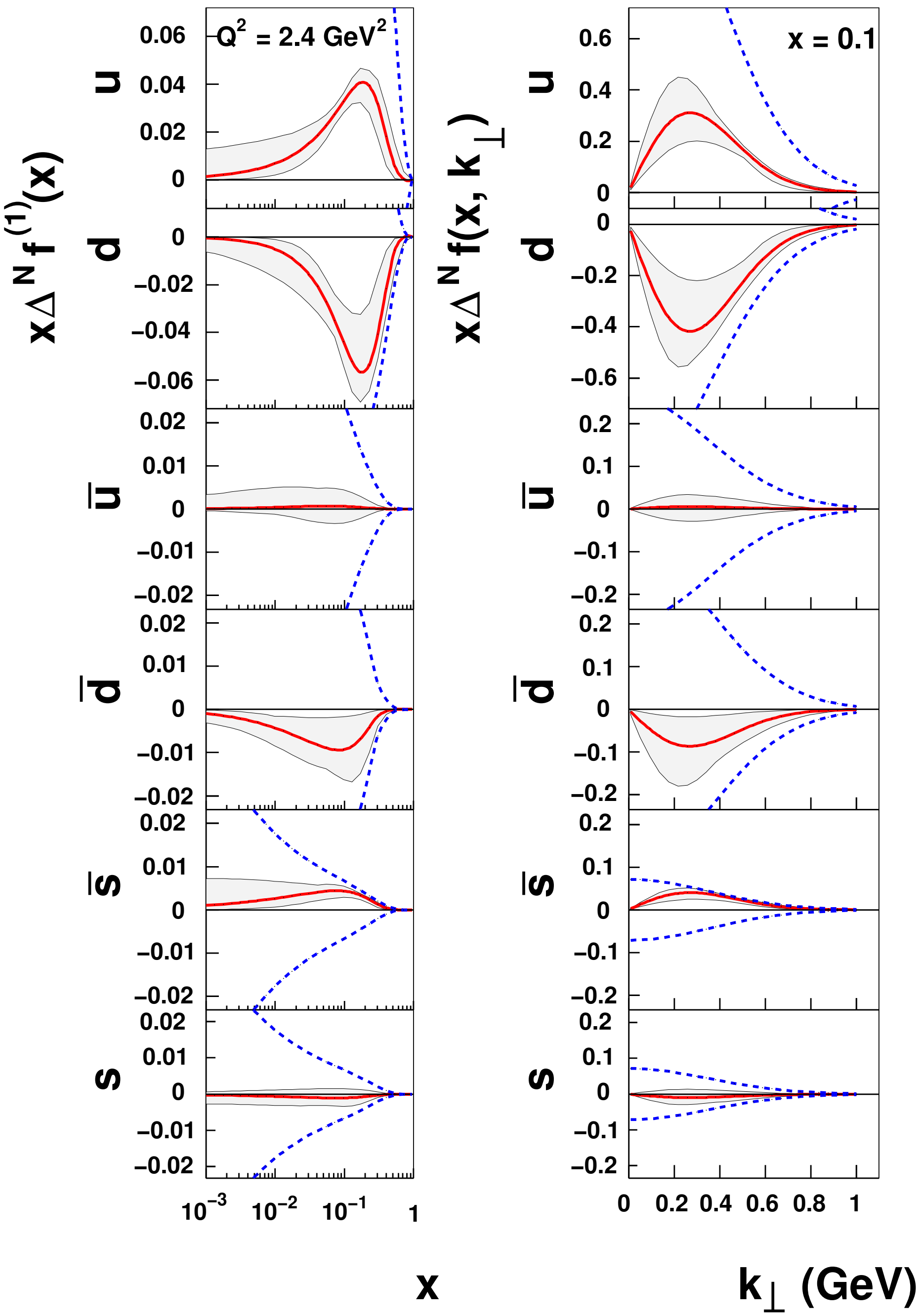




Fit of HERMES data [*Diefenthaler et al. 2006*, *Pappalardo et al. 2008*]

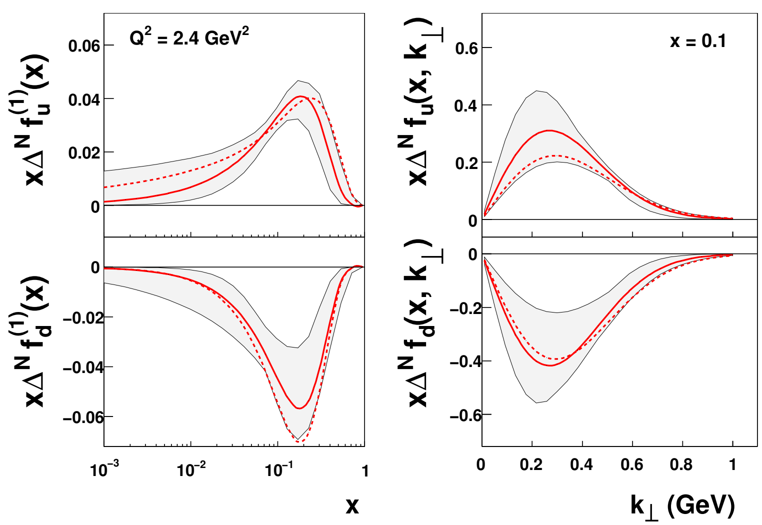
and COMPASS data [*Martin et al. 2006*] (deuteron target)

K_S^0 predicted [not included in the fit]



Sivers functions

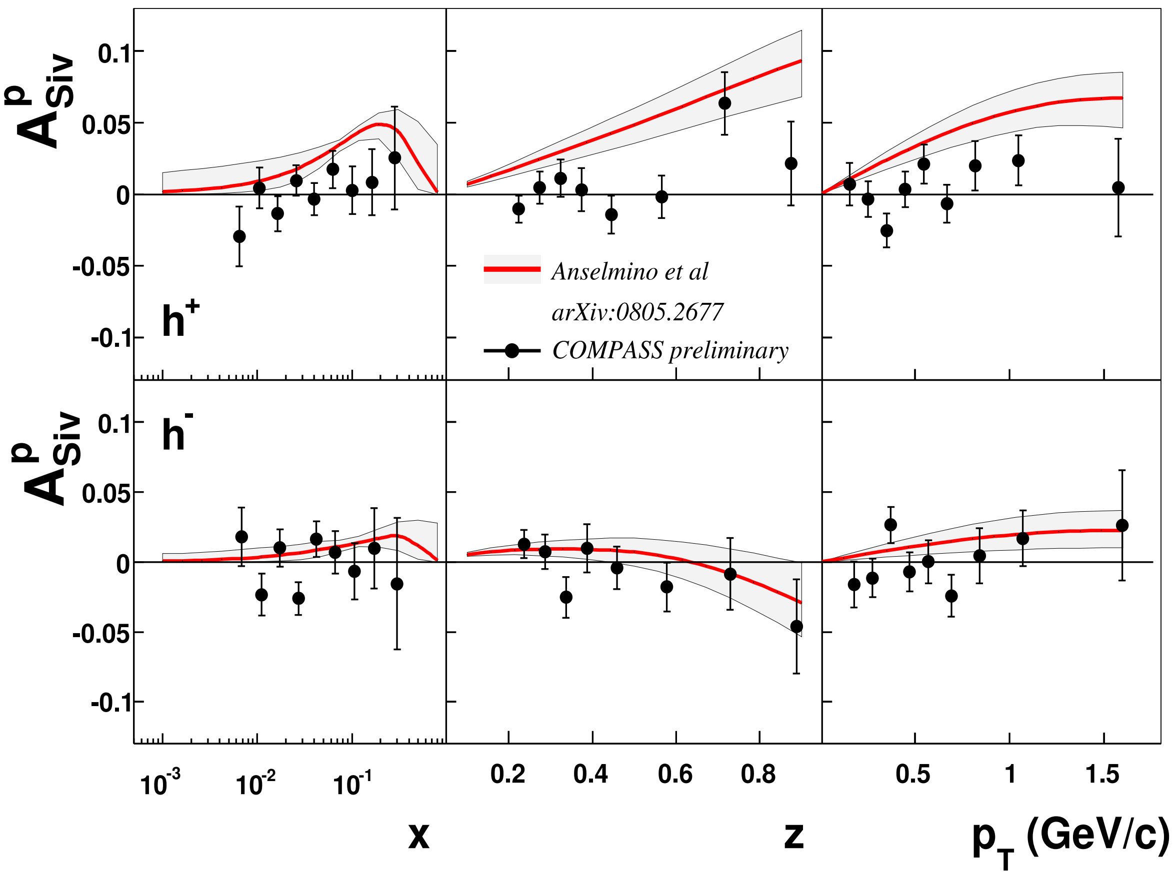
valence quarks: new vs. old fit



$\Delta^N \hat{f}_{u/p^\uparrow} > 0 \quad \Delta^N \hat{f}_{d/p^\uparrow} < 0$

$\Delta^N \hat{f}_{\bar{s}/p^\uparrow} > 0$

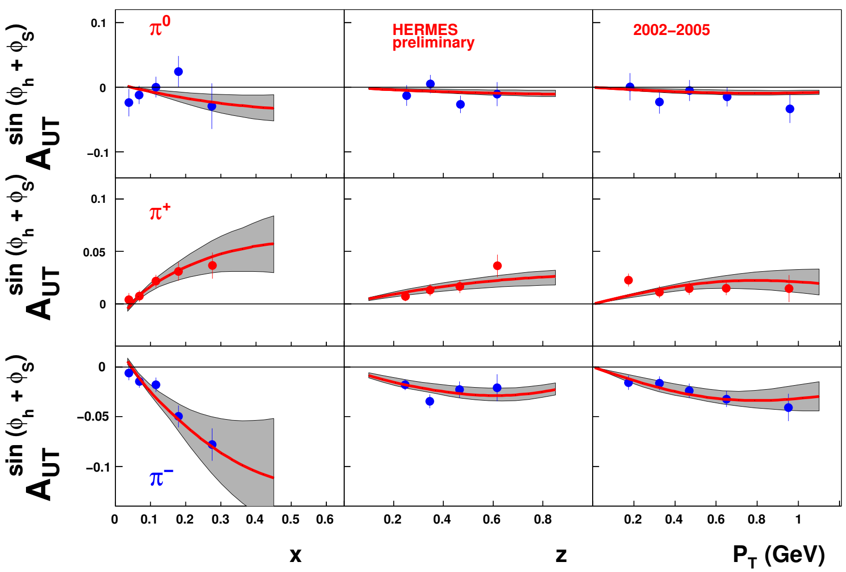
Predictions vs. COMPASS data with proton target (*Levorato 2008*)

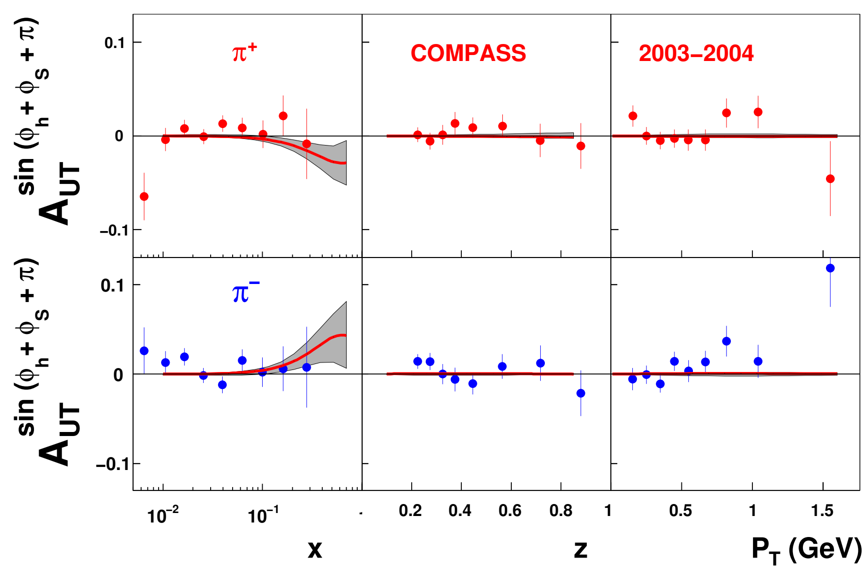


...Controversial.

• Collins effect in SIDIS: Best Fit

Anselmino, Boglione, UD, Kotzinian, Melis, Murgia, Prokudin, Turk, NP Proc. Suppl. 2009

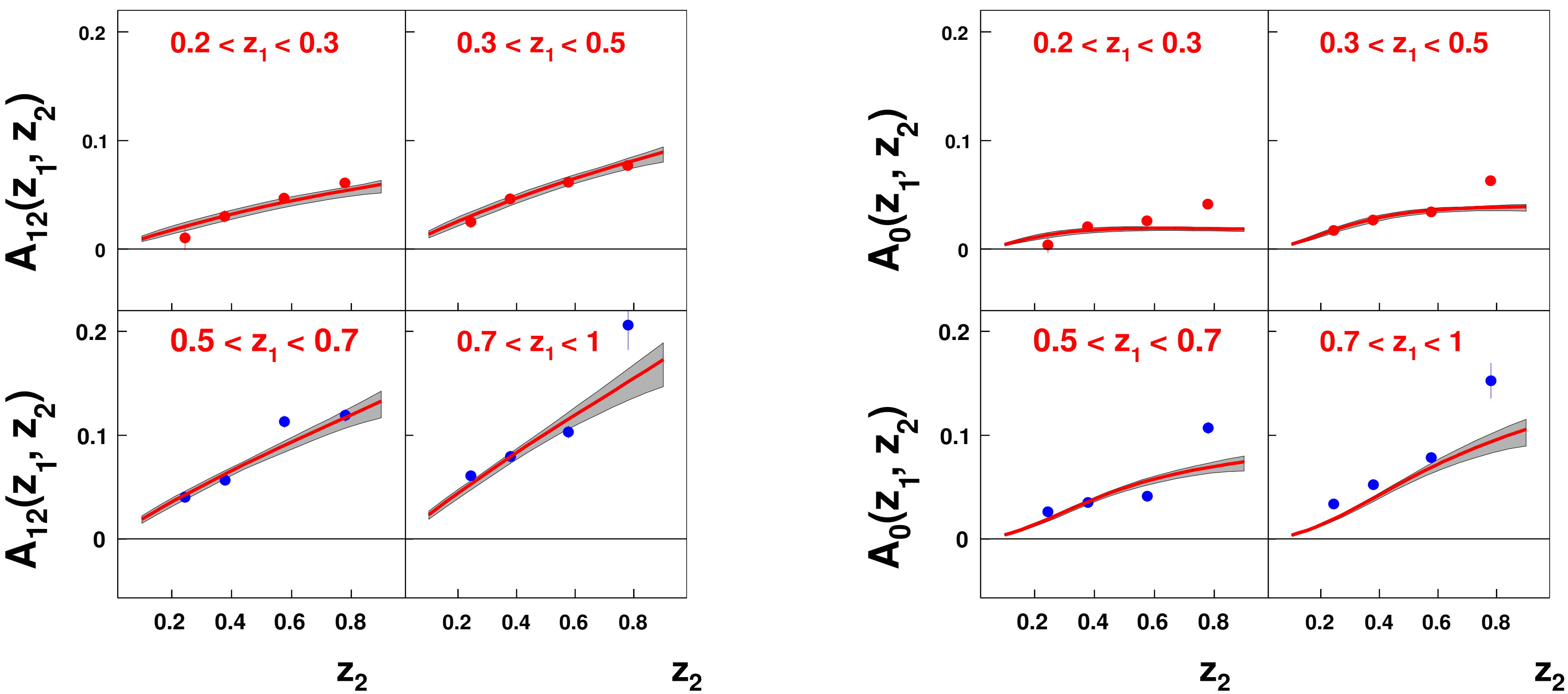




[left] HERMES data [*Diefenthaler et al. 2007*] (hydrogen target)

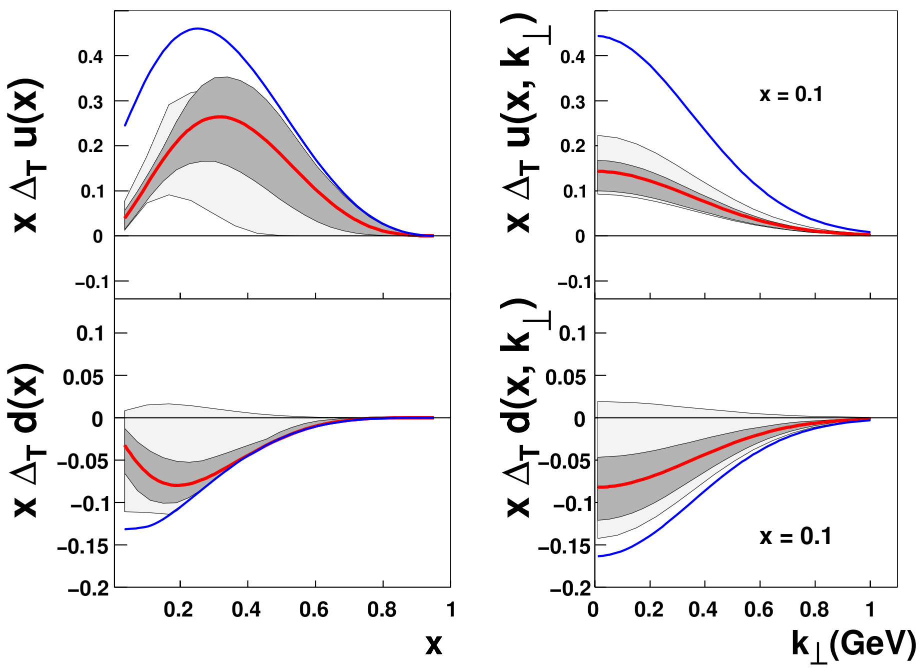
[right] COMPASS data [*Alekseev et al. 2008*]. (deuteron target)

• Collins effect in e^+e^-: Best Fit of Belle data



Fit of A_{12} $\Rightarrow$ comparison with A_0 data. [Belle data, *Seidl et al. 2008*].

Transversity distribution



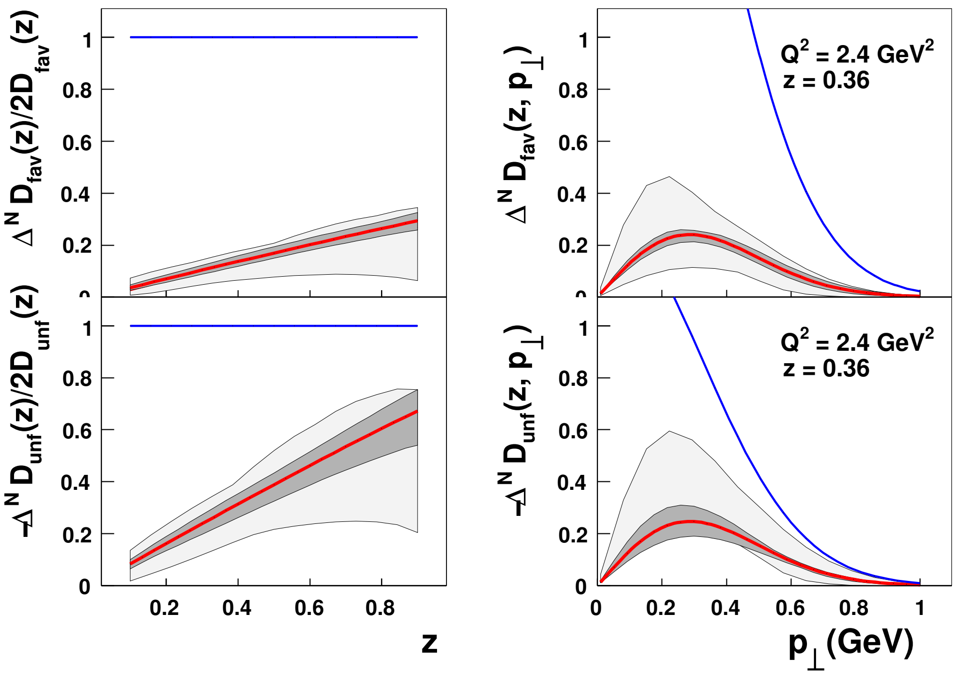
Errors strongly reduced

$\Delta_T u$ larger

sign better constrained

new SIDIS data: CRUCIAL

Collins function



new vs. old error bands

HERMES (proton target)

$A_{UT}^{\pi^+} \simeq 4\Delta_T u\, \Delta^N D_{\rm fav} + \Delta_T d\, \Delta^N D_{\rm unf}$

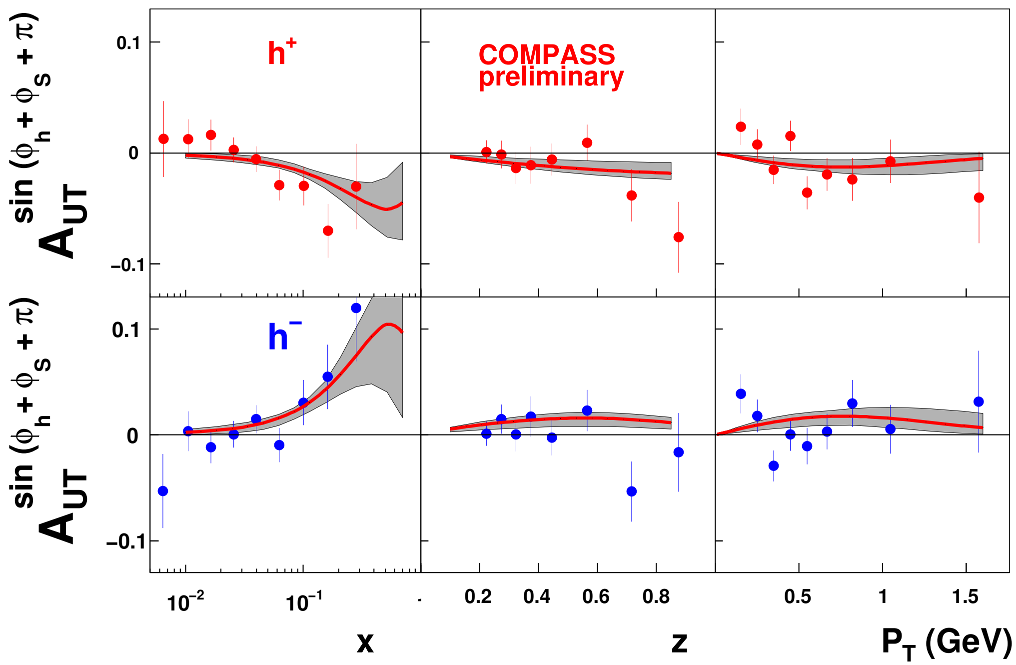
$A_{UT}^{\pi^-} \simeq 4\Delta_T u\, \Delta^N D_{\rm unf} + \Delta_T d\, \Delta^N D_{\rm fav}$

$|A_{UT}^{\pi^-}| > |A_{UT}^{\pi^+}|$

$\Rightarrow$ large and negative unfav. FF

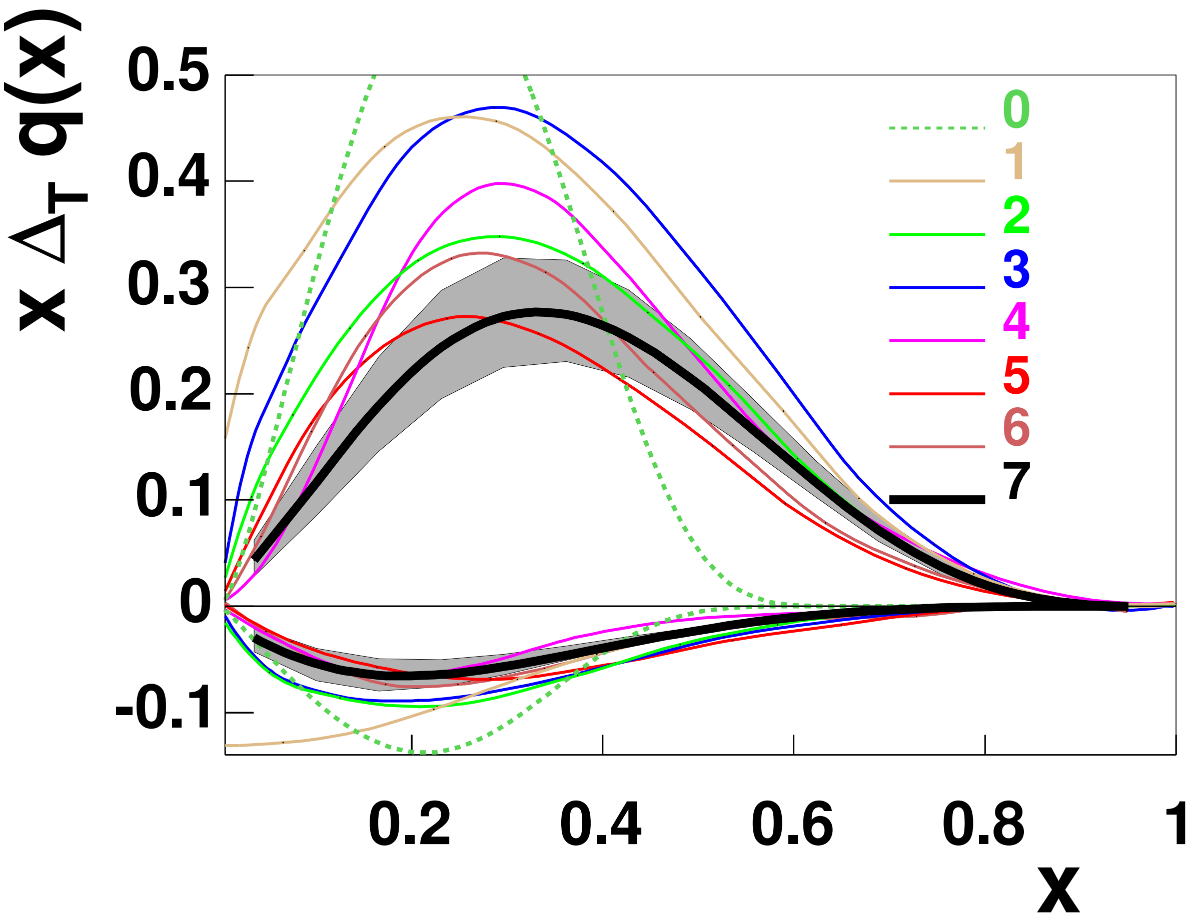
Consistent with other extractions [*Efremov et al. 2006, Vogelsang & Yuan 2005*]

Predictions vs. COMPASS data with proton target (*Levorato 2008*)



Amazing agreement!

Transversity: Comparison with models



0 Barone et al. 1997

1 Soffer et al. 2002

2 Korotkov et al. 2001

3 Schweitzer et al. 2001

4 Wakamatzu 2007

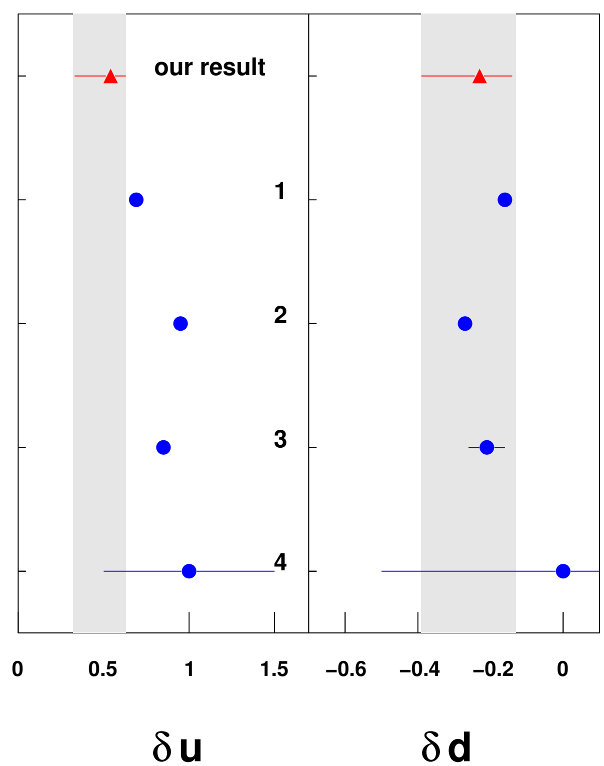
5 Pasquini et al. 2005

6 Cloet et al. 2008

7 Our improved analysis

Tensor charge: $\delta q = \int dx(\Delta_T q - \Delta_T \bar{q}) = \int dx \Delta_T q$

$\delta u = 0.54^{+0.09}_{-0.22}$ $\delta d = -0.23^{+0.09}_{-0.16}$ at $Q^2 = 0.8$ GeV^2



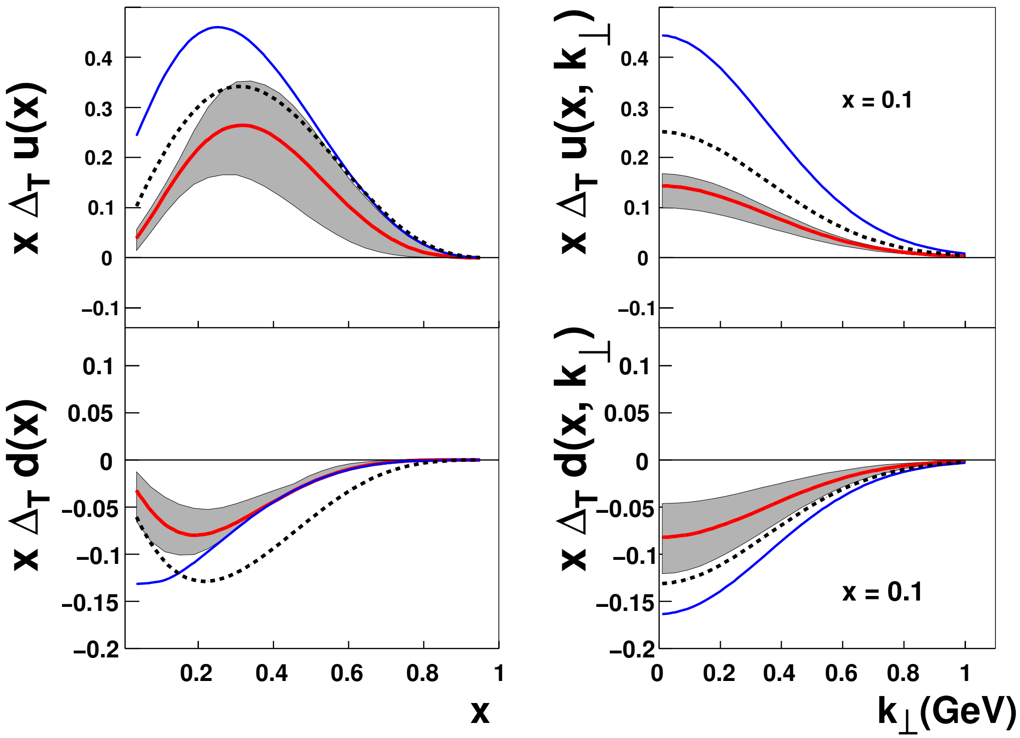
1 Quark-diquark model: *Cloet et al. 2008*

2 CQSM: *Wakamatzu 2007*

3 Lattice QCD: *Goeckeler et al. 2005*

4 QCD sum rules: *He & Ji 1995*

Transversity vs. helicity distribution



transversity: $Q^2 = 2.4\ \mathrm{GeV}^2$

Soffer bound: $(q + \Delta q)/2$

helicity distribution: Δq [GRSV2000]

$|\Delta_T q| < |\Delta q|$: relativistic effect

$\Rightarrow \Delta q - \Delta_T q = \frac{k_\perp^2}{2M^2} h_{1T}^{\perp q}$

in *no-gluon models*

[*Avakian, Efremov, Meissner, Pasquini,...*]

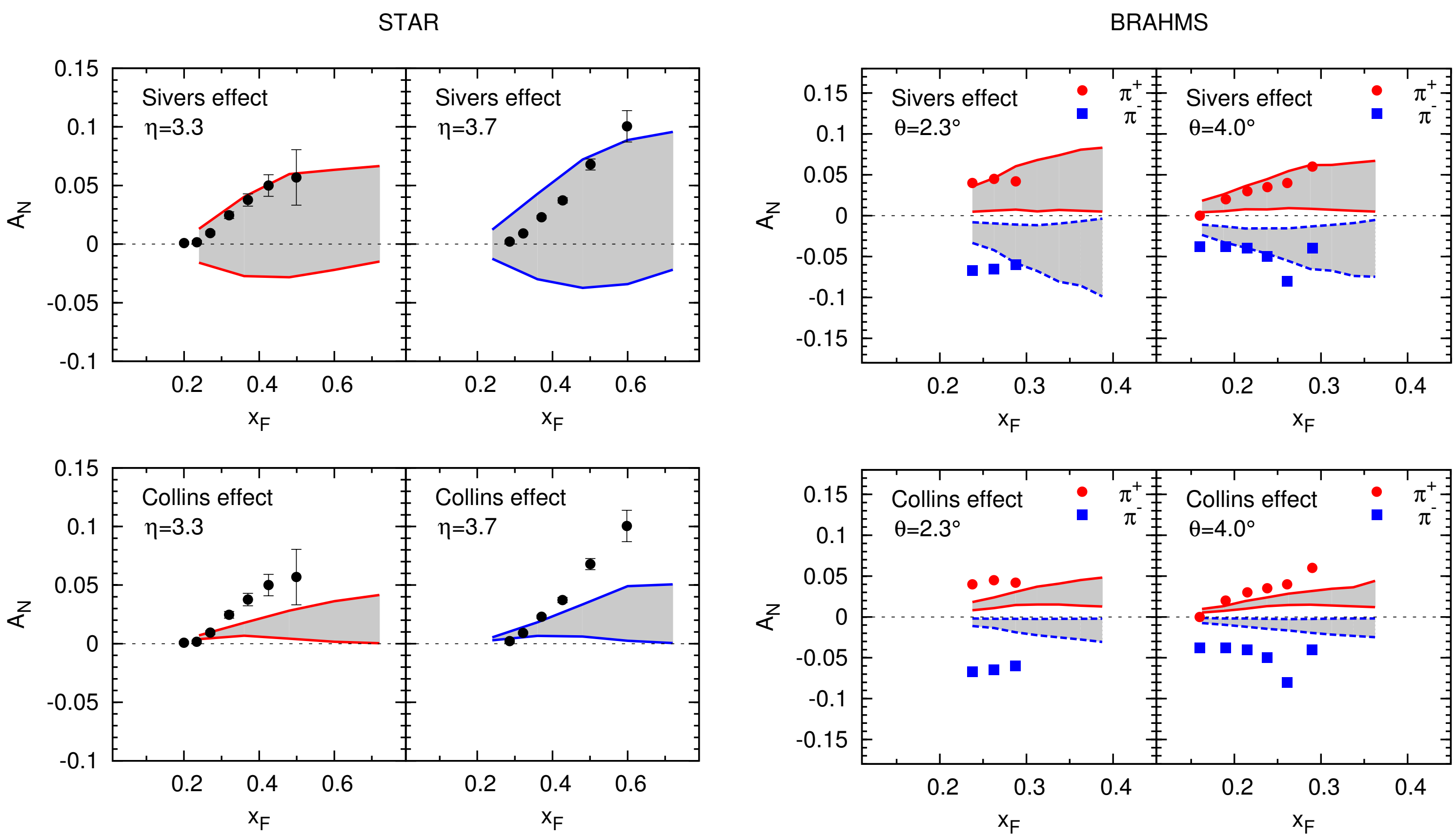
A look at SSAs in $p^{\uparrow} p \longrightarrow \pi X$

Notice:

- competing (*or related*) mechanism: higher-twist terms [*Qiu-Sterman*]

- TMD factorization: *phenomenological assumption* [*Anselmino et al.*]
- many terms but only **two** significant

$$\Rightarrow A_N \simeq A_N^{\text{Sivers}} + A_N^{\text{Collins}}$$



Attempt to describe A_N by scanning the large x region of the Sivers and transversity functions and fitting SIDIS data with $\Delta\chi^2/\chi^2_{\rm min} \leq 20\%$.

- Crucial role of gluon FF in unpol. cross sections

Conclusions

- Global fitting of TMDs has started

- **Sivers functions**

valence quarks: good; sea quarks: first insight
HERMES K^+ data (improvement, but...);
COMPASS proton data?

- **Transversity and Collins functions**

$\Delta_T u$ and $\Delta_T d$ (SIDIS data crucial);
fav/unf. Collins functions (Belle data crucial)
COMPASS proton data!

Open issues

Theory side

- parameterizations: Gaussian $x(z) - k_\perp$ factorization
- evolution: $\Delta^N D_{h/q^\uparrow}(z, Q^2) \leftrightarrow H_1(z, Q^2)$ *impact on* h_1 [in progress]
- soft factor
- A_N in $pp \to h\, X$ *extended global fit* [in progress]

Experimental side

- Correlation matrix errors
- binning
- large x (JLAB)
- $p_\perp$ dependence (Belle)
- A_N and A_{TT} in Drell-Yan processes

Open issues

Theory side

- parameterizations: Gaussian $x(z) - k_\perp$ factorization
- evolution: $\Delta^N D_{h/q^\uparrow}(z, Q^2) \leftrightarrow H_1(z, Q^2)$ *impact on* h_1 [in progress]
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Experimental side

- Correlation matrix errors
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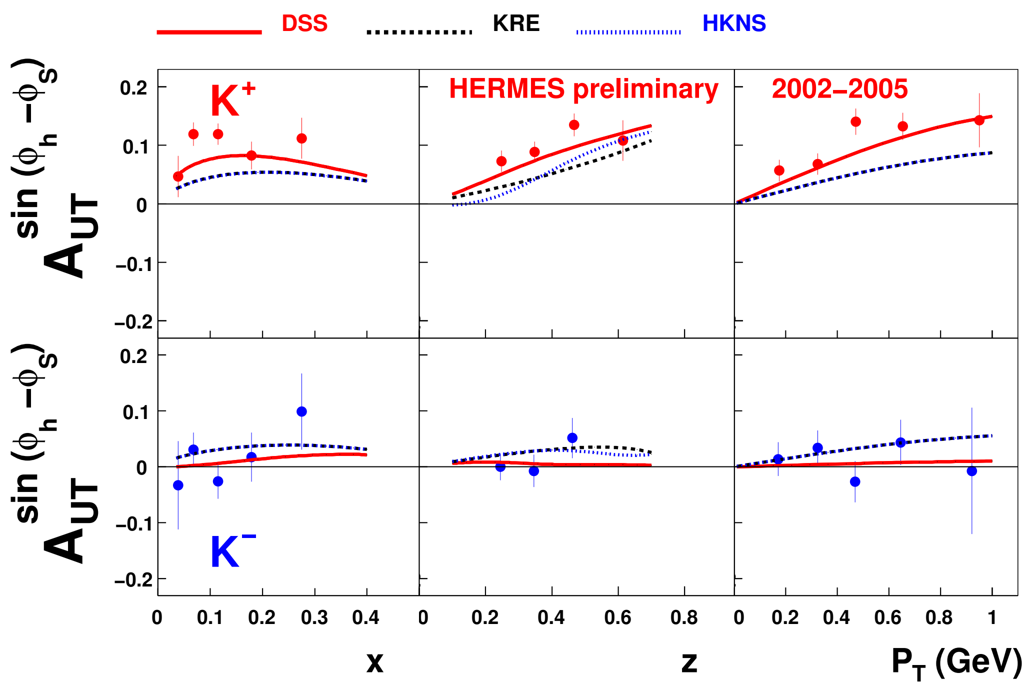
MANY THANKS

BACK-UP SLIDES

Sivers effect for kaons and role of FF sets



Burkardt sum rule

Burkardt PRD69 (2004)

$$\sum_a \int dx\, d^2\boldsymbol{k}_\perp\, \boldsymbol{k}_\perp\, \hat{f}_{a/p^\uparrow}(x, \boldsymbol{k}_\perp) \equiv \sum_a \langle \boldsymbol{k}_\perp^a \rangle = 0$$

$$\langle \boldsymbol{k}_\perp^a \rangle = \left[\frac{\pi}{2} \int_0^1 dx \int_0^\infty dk_\perp\, k_\perp^2\, \Delta^N f_{a/p^\uparrow}(x, k_\perp) \right] (\boldsymbol{S} \times \hat{\boldsymbol{P}})$$

almost saturated by u and d quarks alone at $Q^2 = 2.4\ \mathrm{GeV}^2$:

$$\langle k_\perp^u \rangle + \langle k_\perp^d \rangle = -17^{+37}_{-55}\ \ (\mathrm{MeV}) \qquad \langle k_\perp^{\bar{u}} \rangle + \langle k_\perp^{\bar{d}} \rangle + \langle k_\perp^{s} \rangle + \langle k_\perp^{\bar{s}} \rangle = -14^{+43}_{-66} (\mathrm{MeV})$$

$$\langle k_\perp^u \rangle = 96^{+60}_{-28}\ \ (\mathrm{MeV}) \qquad \langle k_\perp^d \rangle = -113^{+45}_{-51}\ \ (\mathrm{MeV})$$

leaving little room for a gluon Sivers function,

$$-10 \leq \langle k_\perp^g \rangle \leq 48\ (\mathrm{MeV})$$

SIDISLAND

Polarized SIDIS cross section up to subleading order in $1/Q$

$$
\begin{aligned}
d\sigma \quad = \quad & d\sigma^0_{UU} + \cos 2\phi_h \, d\sigma^1_{UU} + \frac{1}{Q} \cos\phi_h \, d\sigma^2_{UU} + \lambda_e \frac{1}{Q} \sin\phi_h \, d\sigma^3_{LU} \\
+ \quad & S_L \Big\{ \sin 2\phi_h \, d\sigma^4_{UL} + \frac{1}{Q} \sin\phi_h \, d\sigma^5_{UL} + \lambda_e \Big[d\sigma^6_{LL} + \frac{1}{Q} \cos\phi_h \, d\sigma^7_{LL} \Big] \Big\} \\
+ \quad & S_T \Big\{ \boldsymbol{\sin(\phi_h - \phi_S) \, d\sigma^8_{UT}} + \boldsymbol{\sin(\phi_h + \phi_S) \, d\sigma^9_{UT}} + \sin(3\phi_h - \phi_S) \, d\sigma^{10}_{UT} \\
& + \frac{1}{Q} \Big[\sin(2\phi_h - \phi_S) \, d\sigma^{11}_{UT} + \sin\phi_S \, d\sigma^{12}_{UT} \Big] \\
& + \lambda_e \Big[\cos(\phi_h - \phi_S) \, d\sigma^{13}_{LT} + \frac{1}{Q} \big(\cos\phi_S \, d\sigma^{14}_{LT} + \cos(2\phi_h - \phi_S) \, d\sigma^{15}_{LT} \big) \Big] \Big\}
\end{aligned}
$$

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